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Evaluation of LC Fresnel Phase Plate Utilized as Colour Filter

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A method to decompose the white light into three primary colours was proposed. The switchable Fresnel Phase Plate (FPP) containing liquid crystal could be utilized as colour filters to form sub-pixels. Some optical properties of such device were estimated by means of Kirchhoff diffraction equation which is scale wave equation derived from Helmholtz's equation. The performance of LC-FPP and conventional colour filter was compared. Other configuration of LC-FPP was concerned. And some experiments were done to verify the theoretical calculation.

Keywords Colour filter; focal length; fresnel phase plate; kirchhoff diffraction equation; liquid crystal

1. Introduction

Nowadays, Liquid crystal display (LCD) still dominates the market for its cheap cost, portable size, light weight, and sophisticated manufacture. It is the most widely used display device in computer, communication, and consumer electronics. Conventional colour filters consisting of dyed material are still used in LCD system, which generates visible colours by transmitting the desired bandwidth of light and absorbing the undesired spectra [1]. However, such kind of colour filter is costly and very power consuming, and its efficiency is not satisfactory. Optical system for display with conventional colour filters has the total propagation only 5–10% of the input light [2]. Many scientists and engineers dedicated themselves to improving the efficiency of the display. Some of them tried to invent a wavelength-selective optical system by using stacks of LC retardation layers [3,4], some of them turned to the diffractive optical system. Hui-Hsiung Lin et al. designed a diffractive optical system containing collimated LED backlight, blazed grating and aspheric lenticular lens array and they claimed that the colour gamut of such device was 73% in the NTSC ratio [1]. Yoichi Taira et al. proposed a diffractive optical device consisting of light guide, diffractive grating and lenticular lens array [5–8]. Il-yong Jung et al. invented an optical device in order to remove the conventional colour filters in LCD, and such device was composed of light guide, colour separation sheet and lenticular lens array [9,10]. Tomohiko

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Mastsushita et al. reported on their invention which was a colour separation element comprising a prism array, two diffraction gratings and light-conducting plate [11].

As we filed a patent of DLCD utilizing LC diffraction grating as colour filter [12] and reported on the FrenLCD based on Fresnel Zone Plate (FZP) [13], we planned to continue our work in making some improvements for our previously proposed optical device. FZP is well known device for focusing light [14]. A Fresnel Zone Plate consists of a set of radially symmetric rings, known as Fresnel zones, which alternate between opaque and transparent. Light hitting the zone plate will diffract around the opaque zones. The zones can be spaced so that the diffracted light constructively interferes at the desired focus, creating an image there. Because it can be made into thin lens and it is very easy to manufacture, FZP is widely used. However, with its alternatively opaque and transparent structure, the limitation of such device applied in LCD is obvious: low transmittance and low efficiency. A Fresnel Phase Plate which provides constructive interference at the focus by means of phase interference can offer high transmittance, thus win the advantage over FZP. Conventionally, Fresnel Phase Plate is transparent and is made up of an isotropic material with some rectangular protrusion structures to offer extra optical paths [15]. In the field of liquid crystal, the configuration of FPP can be different from the conventional one. By using the birefringence of liquid crystal, one can get the structure with two refractive indices n_e and n_o alternatively aligned. When light propagates through the LC-FPP which has a uniform thickness, two adjacent zones with different refractive indices generate the optical path difference which is equal to half of the wavelength. Thus, the focusing effect of LC-FPP is very easy to achieve. Fresnel lenses have found many important applications in photonics, optical imaging, long distance optical communication, projection displays, space navigation and etc.[16,17] Therefore, the design for LC lens becomes a hot topic. The switchable LC-FPP is promising, and it can be very useful in many fields.

In this article, we aimed at designing an LC-FPP optical device for display at the very beginning. We used Kirchhoff diffraction equation to estimate the optical system in a theoretical way. And we investigated several optical properties of LC-FPP and considered the possibility of utilizing it as colour filter in LCD. We also considered the case of asymmetric Semi-LC-FPP, which was exactly half of the symmetric LC-FPP, and we compared both cases. Furthermore, we did some experiments to verify our theoretical calculation. In our experiment, we used homogeneous cells and only considered the pure diffraction colour which was not influenced by the birefringence colour. Because of some limitations in the experiment, the results might not be very impressive, but some of them coincided with the theoretical prediction. To get a wide range of colour from LC-FPP or Semi-LC-FPP, we needed to optimize some parameters and improve the optical device.

2. Design

To use LC-FPP as colour filter in an LCD, collimated and polarized backlight of CCFL or RGB LED source was required, as shown in Fig. 1.

In our LC-FPP, we used ECB mode to control the liquid crystal with the initially homogeneous alignment. In the theoretical part, we used common liquid crystal E7 for discussion. As shown in Fig. 2, a polarizer was put along the substrate (X axis), and the substrates were coated with patterned ITO. Each LC-FPP unit could form one sub-pixel,

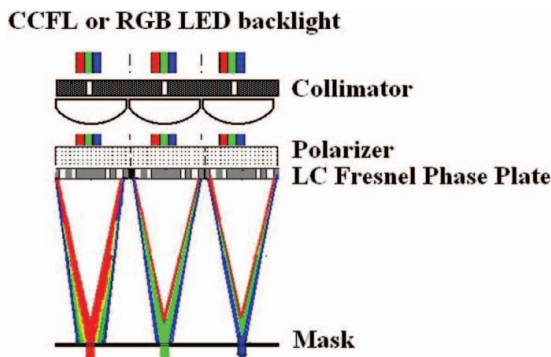


Figure 1. Schematic of optical system containing LC Fresnel Phase Plate.

and each specified wavelength (Red, Green or Blue) corresponded to a specific ITO pattern. The ITO pattern followed the equation: $(R_m)^2 = m\lambda f$, where R_m was the radius of the m th zone, $m = 1, 2, 3, \dots$, λ was the expected wavelength, f was the focal length. The thickness d of the LC-FPP satisfied the equation: $n_e d - n_o d = \lambda/2$.

When voltage was applied, the liquid crystal molecules between the electrodes would rotate to a homeotropic direction, forming some areas that had the refractive index of n_o . Meanwhile, the rest areas remained the refractive index of n_e . As the polarized input light shone on the LC-FPP, the light with the desired wavelength, for instance, $\lambda_R = 650$ nm, would be focused at the focal distance f_R on the optical axis (Z axis). It was similar to get the focused light when we used $\lambda_G = 550$ nm or $\lambda_B = 450$ nm. If we set $f_R = f_G = f_B = f_{com}$ and put a mask with a slit at the focal distance f_{com} , we could get three sub-pixels simultaneously. Because the slit width determined the monochromatic characteristics of

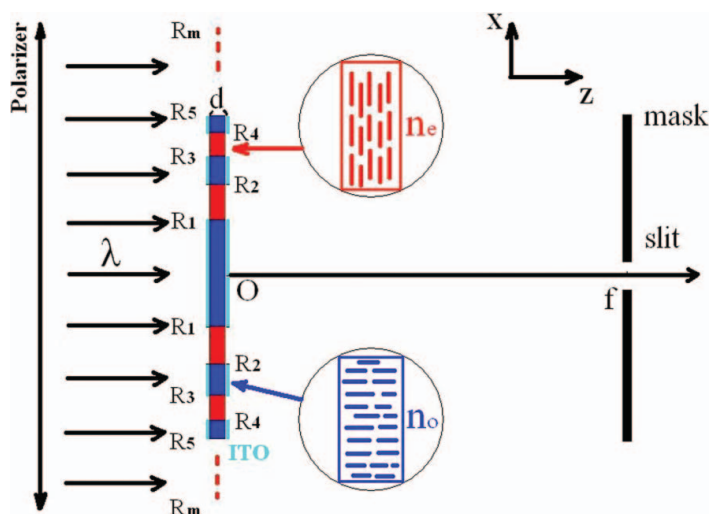


Figure 2. Schematic of one unit of LC Fresnel Phase Plate.

the device and it could not be too narrow, which would reduce the transmittance and thus decrease the efficiency, we needed to optimize it.

3. Theoretical Analysis

Because the polarization of the input light was along the principal axis of either homogeneous or homeotropic liquid crystal in between the two substrates, the polarization of the output light will not change. Moreover, the focal length of the LC-FPP $f_{\text{com}} = 4.2$ mm, and the size of each LC-FPP unit was about $300 \mu\text{m}$, which was much smaller as compared to 4.2 mm. In this case, we used Kirchhoff diffraction equation, which is the scale wave equation derived from Helmholtz's equation: $\nabla^2 E + k^2 E = 0$, to estimate the optical properties of LC-FPP. As shown in Fig. 3, we used 2D equation in x-z plane:

$$E(p) = \frac{1}{i\lambda} \int_{\Sigma} A(\xi) \frac{\exp(ikr)}{r} d\xi \quad (1)$$

where λ was the wavelength of the incident light, Σ was the line where the optical device was located, $k = \frac{2\pi}{\lambda}$, $r = \sqrt{(\xi - x)^2 + d^2}$, $A(\xi) = B(\xi)T(\xi)$, and $T(\xi)$ was a transmission coefficient, $T(\xi) = \frac{|T(\xi)| \exp[i\Phi_T(\xi)]}{\exp[i\Phi_T(\xi)]}$, $\Phi_T(\xi)$ depended on the refractive index of optical device. It meant that the complex amplitude of electric field at any spot $p(x)$ on line Π came from the contribution of every spot of the optical device (line Σ).

In this theoretical section, we set $f = f_{\text{com}} = 4.2$ mm, $\lambda_R = 650$ nm, $\lambda_G = 550$ nm, $\lambda_B = 450$ nm, $n_e = 1.7472$, $n_o = 1.5217$, the thickness of the LC-FPP $d = \lambda/2(n_e - n_o)$, phase retardation $\Phi_T(\xi) = -\frac{2\pi nd}{\lambda}$, $R_{m\text{max}} = 151 \mu\text{m}$ ($m = 12$). And we assumed that 100% light propagated through the LC Fresnel Phase Plate without reflection and loss, thus $|T(\xi)| = 1$.

Because the intensity $I \propto E \cdot E^*$, we could get the intensity distribution along the line Π (see Fig. 4). If we put a mask with a slit on the line Π , it would block some portion of the light. The ratio $I(\lambda) = I_{\text{output}}(\lambda)/I_{\text{input}}(\lambda)$ was just the transmittance which indicated that how much light of the wavelength λ remained after passing through the slit. If we knew the slit width, the intensity of the output light could be easily calculated.

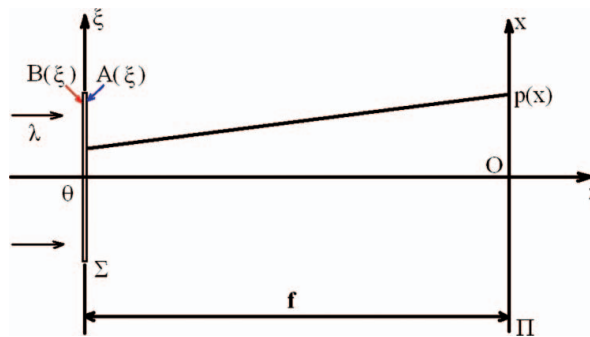


Figure 3. Schematic of Kirchhoff 2D diffraction equation. $B(\xi)$ and $A(\xi)$ were complex amplitude of electric field for the front surface and back surface of the optical device respectively. $p(x)$ was an arbitrary spot on the line Π located at the distance f from the line Σ .

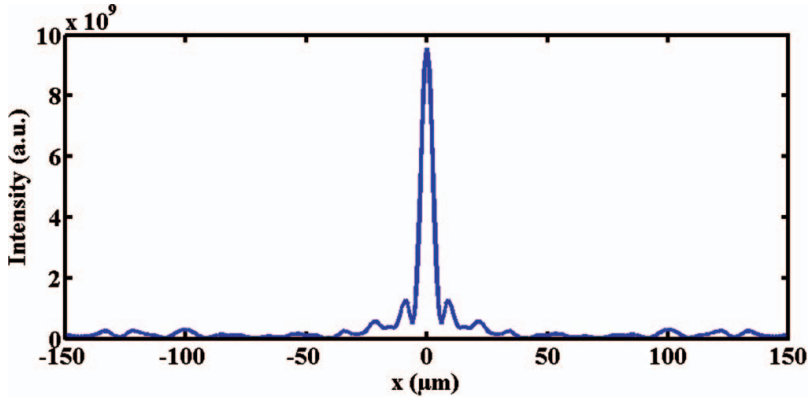


Figure 4. Light intensity distribution along x axis using 450 nm light source.

We used common light source D65, of which the wavelength λ was from 360 nm to 830 nm in the calculation. We set the width of the slit $w = 50, 40, 30, 20, 10, 5, 1 \mu\text{m}$. For each width of the slit, we obtained the spectrum (see Fig. 5) from the LC-FPP for λ_R , λ_G and λ_B respectively.

According to the tri-stimulus equation of CIE 1931:

$$X = \int I_{\text{input}}(\lambda) I(\lambda) x(\lambda) d\lambda, Y = \int I_{\text{input}}(\lambda) I(\lambda) y(\lambda) d\lambda, Z = \int I_{\text{input}}(\lambda) I(\lambda) z(\lambda) d\lambda \quad (2)$$

where $x(\lambda)$, $y(\lambda)$ and $z(\lambda)$ were human eye sensitive functions, we obtained the values of $x = X/(X+Y+Z)$, $y = Y/(X+Y+Z)$. Subsequently, we got the colour triangle of three sub-pixels (RGB) from the optical system containing three LC-FPP units. As shown in Fig. 6, we plotted seven colour triangles for various slit widths in CIE x-y space and compared them with the colour triangle of NTSC. The largest colour triangle occurred when the slit width was $1 \mu\text{m}$. It was about 37.98% in the NTSC ratio, which was comparable to 38% of an optical device containing transmission-type diffraction grating film [18] and was lower than 42% of a 13.3-type colour filter direct viewing liquid crystal display unit. However,

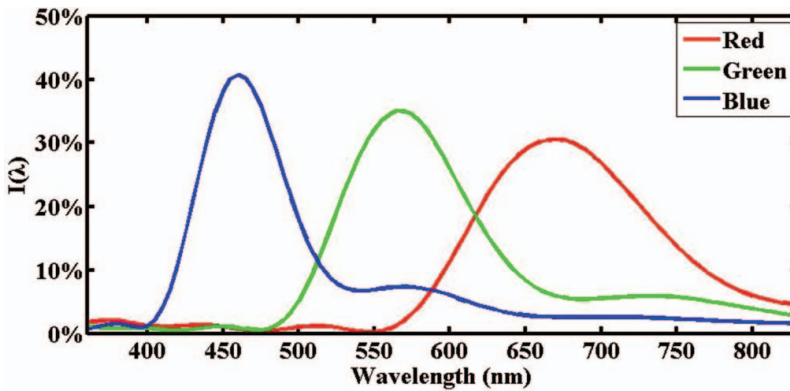


Figure 5. RGB spectra illustrating $I(\lambda)$ versus wavelength. The slit width for this figure was $5 \mu\text{m}$.

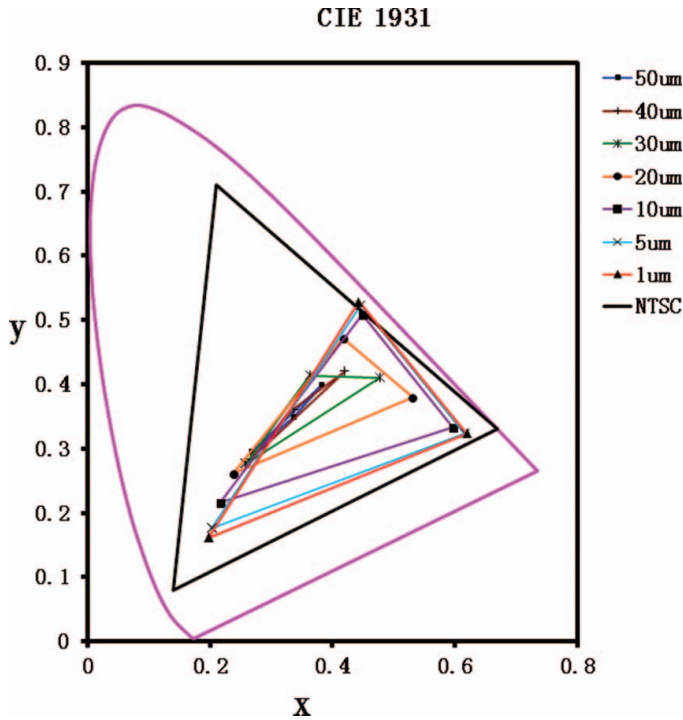


Figure 6. Colour triangles for various widths of slit. The black one represented NTSC.

it sacrificed too much transmittance and efficiency. The optimal option was to choose the slit width equal to $5\ \mu\text{m}$. The colour triangle for the slit width of $5\ \mu\text{m}$ was 35.58% in the NTSC ratio and the transmittance was around 30%. As shown in Fig. 6, the red colour and blue colour from the LC-FPP were almost acceptable though the green colour was very poor leading to a small colour triangle. Because the spectrum for green light was a little bit too broad, it was difficult to get pure green colour. If we want to use such kind of optical system for display, we should optimize the optical device for green colour.

As shown in Fig. 7, we also got the tendency of the transmittance and area of colour triangle in the NTSC ratio with the slit width decreasing. If we want to use this optical device to substitute for conventional colour filter, the transmittance of the light should be much larger than 30%, because conventional colour filter will absorb 70% of the light, and the colour triangle should not be too small as compared to NTSC. In this case, we found that the performance of the LC-FPP was not satisfactory, so that it was difficult to replace the conventional colour filter. But the process of producing LC-FPP is very easy and the cost is much cheaper as compared to colour filter. The optical device only consists of a polarizer, two substrates coated with patterned ITO, a liquid crystal layer and an output mask. Although LC-FPP does not meet the criterion of replacing colour filter for display, it can be used in some other fields, for instance, LC Fresnel micro-lens.

One factor that we should also take into account was the divergence of the optical device. If we changed the incident angle of the light, the light intensity distribution would also be changed. We varied the incident angle of the light from 0 to 2 degrees in 0.5 degree increments, and got the result as shown in Fig. 8. Because the focal distance was 4.2 mm, only 0.5 degree change in the incident angle would shift the peak intensity tremendously.

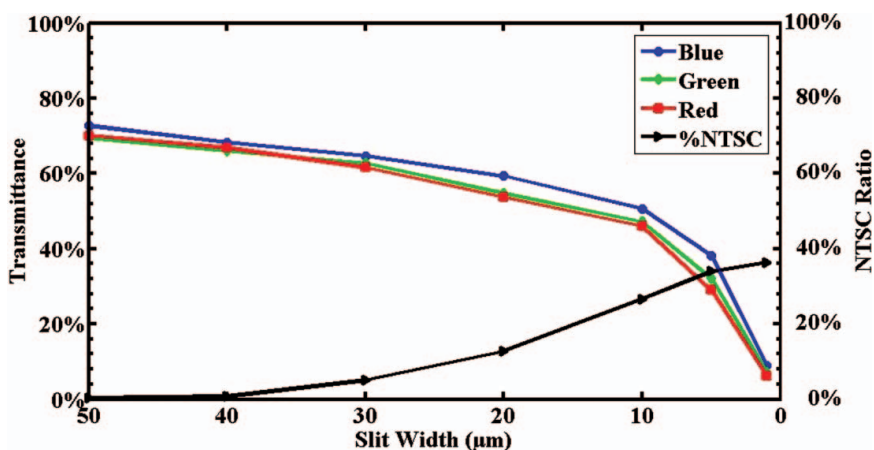


Figure 7. Transmittance versus slit width and NTSC Ratio versus slit width. The red, green and blue curves represented the transmittance of red light, green light and blue light respectively. The black curve represented the colour triangle in the NTSC ratio.

We put an output mask at focal distance with the slit located at the origin ($0 \mu\text{m}$), and the slit width was less than $50 \mu\text{m}$. As a result, the light would be blocked by the mask when the incident angle was larger than 0.5 degree. It meant that the divergence of such optical device would more or less decrease the efficiency of the LC-FPP. In other words, highly collimated light was required for such optical device.

Another configuration of LC-FPP was Semi-LC-FPP (see Fig. 9). We also investigated some optical properties of this kind of LC-FPP and compared it with symmetric LC-FPP.

As shown in Fig. 10, if we cut off half of the symmetric LC-FPP to form a Semi-LC-FPP, the colour triangle got not smaller but a little bit larger. If we doubled the size of Semi-LC-FPP which meant it was the same size as the previous symmetric LC-FPP,

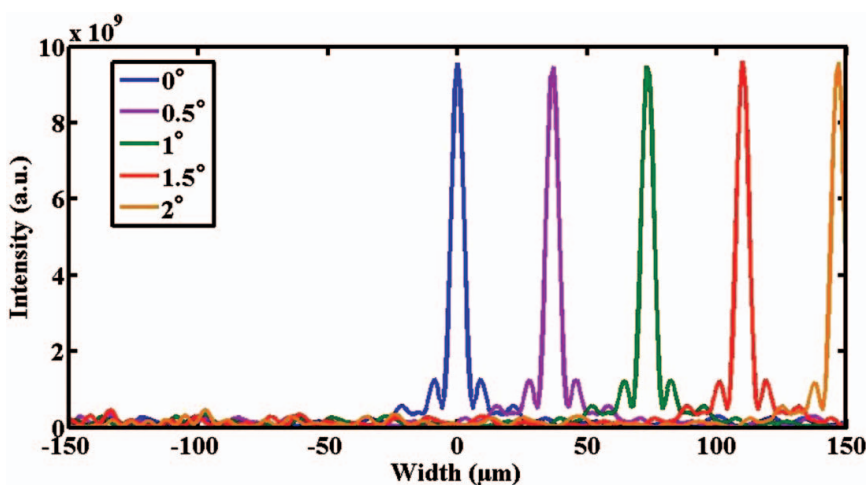


Figure 8. Intensity distributions for various incident angles of the light.

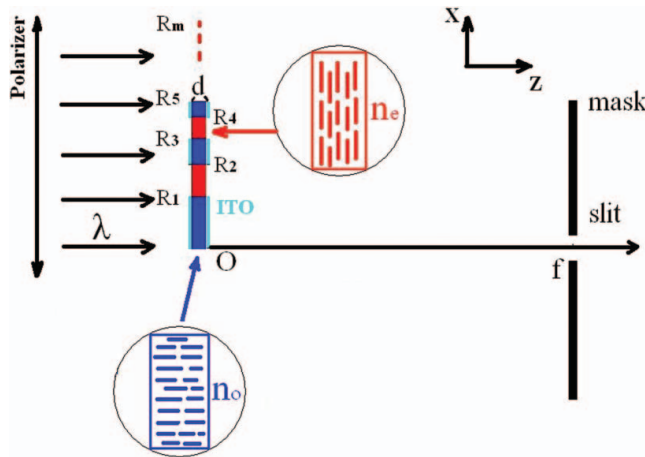


Figure 9. Schematic of one unit of Semi-LC Fresnel Phase Plate.

Semi-LC-FPP would extend its Fresnel zones (R_m) to higher orders (m). Thus, more zones would contribute to the focusing effect and larger colour triangle could be achieved.

By varying focal length of Semi-LC-FPP, we could optimize the focal length to get the largest colour triangle (see Table 1). We found that the colour triangles for $f = 2, 3$ and 4.2 mm were at the same level. Taking divergence into account, $f = 2\text{--}3$ mm was the optimal focal length.

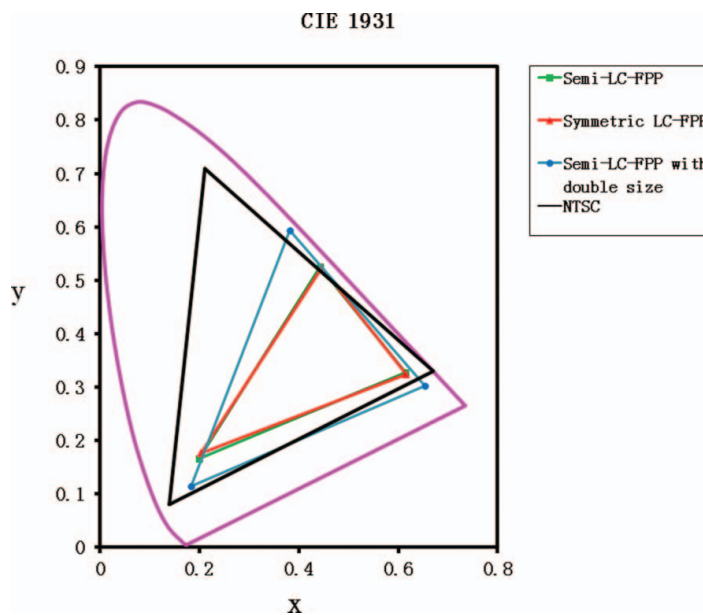


Figure 10. Colour triangles of Semi-LC-FPP and symmetric LC-FPP. The slit width was $5\text{ }\mu\text{m}$. GREEN curve: Semi-LC-FPP with the size of $150\text{ }\mu\text{m}$. RED curve: symmetric LC-FPP with the size of $300\text{ }\mu\text{m}$. BLUE curve: Semi-LC-FPP with the size of $300\text{ }\mu\text{m}$.

Table 1. Colour triangles for different focal lengths in the NTSC ratio

Slit width = 5 μm	$f = 1 \text{ mm}$	$f = 2 \text{ mm}$	$f = 3 \text{ mm}$	$f = 4.2 \text{ mm}$
%NTSC	35.89%	54.04%	51.96%	52.68%

4. Experiment

A simple experiment was arranged to measure the colour triangles for a series of cells. Cells with patterned ITO for different focal lengths were packaged after the process of spin-coating, baking and rubbing alignment layer PI. Because it was not easy to prepare cells with very small cell gap, we chose to use positive liquid crystal MLC-6692 with relatively small $n_e - n_o = 0.085$ to fill in the cell and used different spacers to control the cell gaps. The cell gaps ($d = \lambda/2(n_e - n_o)$) were supposed to be $2.6 \mu\text{m}$ for BLUE (450nm), $3.2 \mu\text{m}$ for GREEN (550nm) and $3.8 \mu\text{m}$ for RED (650nm) in the ideal case. But due to the lack of the spacers mentioned above, we had to use the spacers of $2.5 \mu\text{m}$ for BLUE, $3.2 \mu\text{m}$ for GREEN and $4 \mu\text{m}$ for RED. We made homogeneous cells instead of TN cells. It meant that we could obtain colours only from diffraction and avoid the birefringence colours. The sizes of each unit of the LC-FPP and Semi-LC-FPP were $\sim 300 \mu\text{m}$ and $\sim 150 \mu\text{m}$ respectively.

The set-up of the experiment was shown in Fig. 11. We used a collimator to generate collimated light. Unfortunately, the collimator did not work very well so that the light was not highly collimated. When we applied voltage ($\sim 1\text{--}3\text{V}$) to the substrates of the cell to switch on the Fresnel pattern, light came from behind the cell and then was decomposed into various colours. We observed the colours by using microscope and captured the pictures of rainbow-like stripes of colour.

We compared the colour triangles of symmetric LC-FPP and Semi-LC-FPP. The results were shown in Fig. 12. The Semi-LC-FPP had the same focal length (4.2 mm) as the symmetric LC-FPP. The colour triangle of Semi-LC-FPP was comparable to that of symmetric LC-FPP. It was consistent with what we had predicted. Therefore, Semi-LC-FPP performed better than symmetric LC-FPP when they were the same size.

We also attempted to optimize the focal length of the Semi-LC-FPP. As shown in Fig. 13, though all the colour triangles were very small, it turned out that the tendency still worked. The optimal focal length was 2 mm which we had predicted by the calculation.

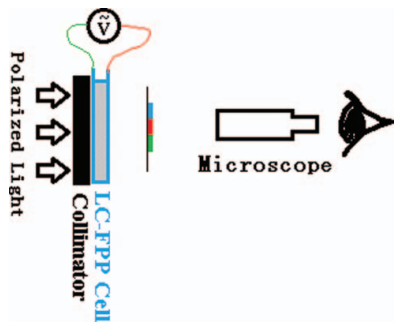


Figure 11. Schematic of the set-up of the experiment.

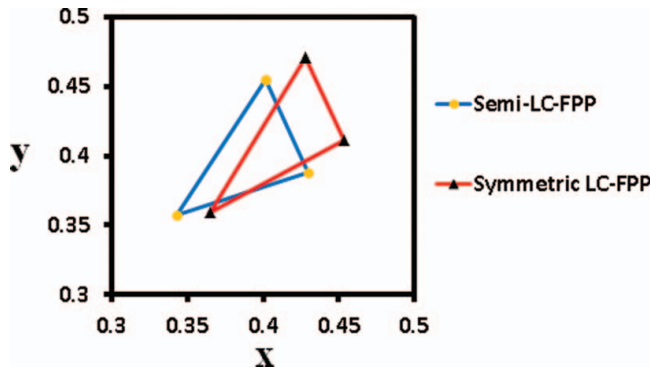


Figure 12. Experimental colour triangles of Semi-LC-FPP and symmetric LC-FPP.

The Semi-LC-FPP with 1 mm focal length had the smallest colour triangle among all these cells.

5. Discussion

Although there are some drawbacks of the current designed LC-FPP, and the performance of such optical device was not satisfactory from the perspective of liquid crystal display, we can make some improvements to the proposed optical device. We have optimized some parameters of this optical system such as focal length, slit width, and we used asymmetric Semi-LC-FPP with larger R_m instead of the symmetric one, but all the colour triangles we obtained were small. Several factors might affect the colours in the experiment. The major influence was from the poorly collimated light. Because of the divergence, less white light was converted to three primary colours. Besides, the alternative spacers made the constructive interference of the expected light weakened and thus lowered the purity of colour. Furthermore, when voltage was applied, liquid crystal might not align in strictly homogeneous or homeotropic directions as we expected, because we used one patterned ITO and one non-patterned ITO glass as the two substrates of the cells. It made the situation more complicated. Also, the incandescent lamp built in the microscope was not white but

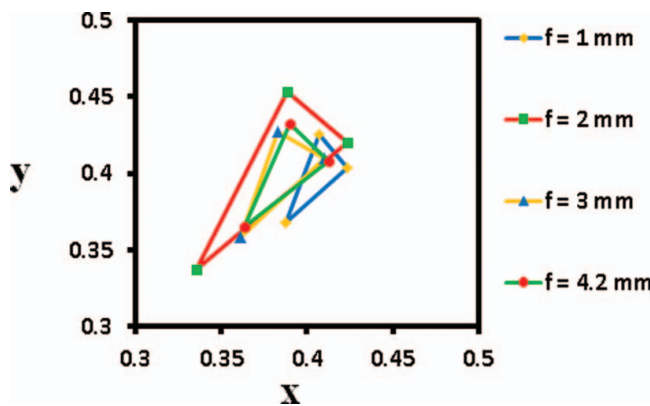


Figure 13. Experimental colour triangles of Semi-LC-FPP ($\lambda = 550$ nm) with different focal lengths.

yellowish, though it was very bright. All these limitations resulted in small colour triangles. If we improve these factors, the colour triangles are sure to be much larger.

Encouragingly, there are some advantages of the proposed optical device. One of them is that the size of such device can be either very small or very large, and it can meet various demands. Another merit of the LC-FPP device is that the focusing effect of such device can be switched on or off by controlling the applied voltage and its focal length can even be tunable by elaborating the design of ITO layers. We may have two options to make the focal length of LC-FPP tunable:

- (1) We may use multilayer structure as shown in Fig. 14(a). Each ITO layer is isolated from one another by coating a SiO₂ layer in between every two ITO layers.
- (2) We may use the discrete structure as shown in Fig. 14(b). i) No voltage is applied to ITO. ii-iv) Voltage is applied to some regions of the discrete ITO structures, forming various patterns to create LC Fresnel Phase Plate with various focal lengths.

As shown in Fig. 15, we have done the experiment to observe the discrete Fresnel Phase Plate. We measured the focal length for the original Fresnel Phase Plate (c). It was about 7 cm. We put the original Fresnel patterned ITO with different focal length (70 mm and 500 mm) onto the discrete structures, of which the pitch was 3 μm . Then we applied voltage (~ 3 V) to the original Fresnel patterned ITO. The discrete Fresnel pattern (d) shown that it was very similar to the original Fresnel pattern (c). Due to the short circuit occurring in some parts of the discrete structure, the third stripe counted from the left of the pattern became broader, but it can be easily improved. (e) and (f) were original and discrete Fresnel pattern for the expected focal length of 500 mm. They were almost the same. Therefore, the discrete structure could help to make the Fresnel Phase Plate tunable as long as we applied voltage to the specific regions of the discrete structure.

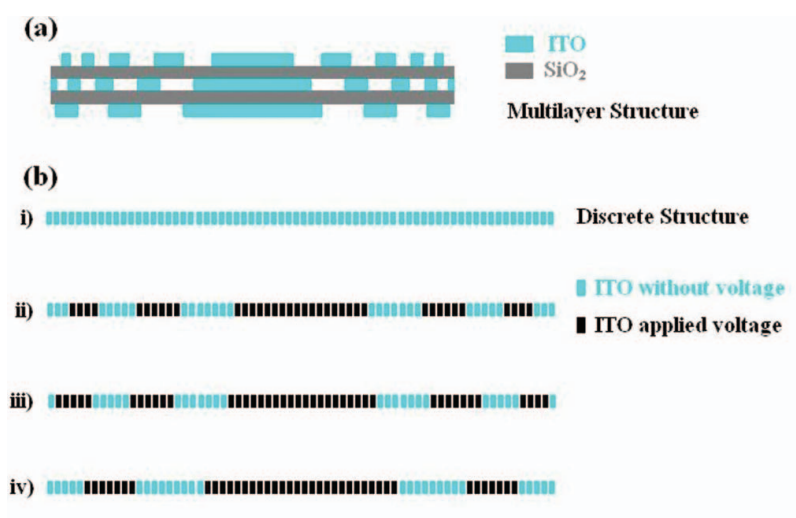


Figure 14. (a) Schematic of multilayer structure (b) Schematic of discrete structure.

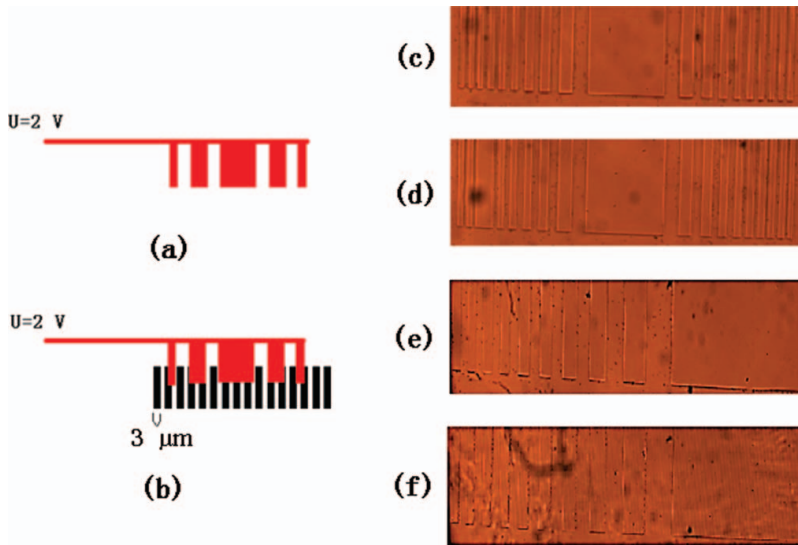


Figure 15. (a) Schematic of original Fresnel ITO pattern. (b) Schematic of discrete ITO structure touched by original Fresnel ITO pattern, pitch = $3\text{ }\mu\text{m}$. (c) Original Fresnel pattern ($f = 70\text{ mm}$). (d) Discrete Fresnel pattern ($f = 70\text{ mm}$). (e) Original Fresnel pattern ($f = 500\text{ mm}$). (f) Discrete Fresnel pattern ($f = 500\text{ mm}$).

6. Conclusion

In the theoretical part, we have studied some optical properties of LC-FPP, including the focusing effect (light intensity distribution along the line \square at the focal distance), the RGB spectra for the red, green and blue light passing through a slit with specific width, the colour triangles for LC-FPP with various slit widths, the transmittance of LC-FPP for RGB light, and the divergence of the optical device. We demonstrated that LC-FPP possessed the capability of light-focusing. However, this configuration of Fresnel Phase Plate shown limited focusing efficiency ($\sim 40.5\%$), so that the purity of the colour at the focal point was not as good as we expected, causing the spectra of the RGB light a little bit too broad, and thus resulted in forming sub-pixels with small colour triangle. We noticed that red and blue colour was somewhat acceptable, though the green colour was very close to yellowish green. Besides the disadvantage of monochromatic characteristic, the efficiency of the designed LC-FPP device was not satisfactory for the sake of relatively low transmittance. Moreover, the divergence of the designed LC-FPP device further decreased the efficiency of the optical system when highly collimated light was not employed. But this negative factor could be optimized by reducing the focal length. Consequently, the current designed LC-FPP was not appropriate to substitute for colour filter in LCD at this stage, though it was capable of separating colour.

Furthermore, we considered Semi-LC-FPP and compared it with the symmetric LC-FPP. Through calculation, we predicted that the colour triangle of Semi-LC-FPP should be comparable to that of symmetric LC-FPP, when the size of symmetric LC-FPP was twice as large as that of Semi-LC-FPP. We also predicted that the optimal focal length should be $2\text{--}3\text{ mm}$. In the experimental part, we have done some experiments to verify these two predictions. The experimental results shown that what we had predicted were correct. However, some limitations in the experiment, for example, poorly collimated light,

alternative spacers, yellowish backlight, made the colour triangles small. If we want to get large colour triangles, we need to improve these factors.

Because it is easy to manufacture an LC-FPP device, the cost of such device is very cheap, and the focusing effect of such device can be switchable and its focal length can be tunable, LC-FPP is possible to have applications in some other fields, for example, LC Fresnel micro-lens for some special experiments, LC Fresnel lens for projectors or projection televisions, LC Fresnel mini-lens for photocopiers or mobile-phone cameras.

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References

- [1] Lin, H. H., Lee, C. H., & Lu, M. H. (2009). *OPTICS EXPRESS*, 17(15), 12397.
- [2] Huang, Y. P., Lin, F. C., & Shieh, H. P. D. (2011). *Journal of Display Technology*, 7(12), 630–632.
- [3] Gray, D. S., & Boulder, C. (1999). Patent US No. 5990996 A.
- [4] Jeroen, B., Tian, H., Pieter, J. M. V., Robert, Z., & Kristiaan, N. (2009). *Mol. Cryst. Liquid Cryst.*, 502, 19–28.
- [5] Yoichi, T., Daiju, N., Hidetoshi, N., Akiko, N., Kuniaki, S., & Fumiaki, Y. (2002). *EURODIS-PLAY 02, P-54*, 585–588.
- [6] Yoichi, T., Hidetoshi, N., Kuniaki, S., & Fumiaki, Y. (2003). *SID 03, P-56*, 340–341.
- [7] Hidetoshi, N., Shinya, O., Fumiaki, Y., & Yoichi, T. (2007). Patent US No. 7164454 B2.
- [8] Yoichi, T., & Daiju, N. (2008). Patent US No. 7315419 B2.
- [9] Jung, I. Y. (2009). Patent US No. 7580083 B2.
- [10] Jung, I. Y. (2010). Patent US No. 7742123 B2.
- [11] Tomohiko, M., Masayuki, S., & Shigeru, A. (2000). Patent US No. 6151166 A.
- [12] Tsvetkov, V. A., Chigrinov, V. G., Kwok, H. S. (2011). Patent US No. 61/457, 244.
- [13] Tsvetkov, V. A. (2006). *Proc. of ASID 06*, 183–184.
- [14] Barrett, H. H., & Horrigan, F. A. (1973). *Applied Optics*, 12(11), 2686–2702.
- [15] Lawrence, A. H. (1987). *Applied Optics*, 26(17), 3649–3654.
- [16] Fan, Y. H., Ren, H. W., & Wu, S. T. (2003). *OPTICS EXPRESS*, 11(23), 3080–3086.
- [17] Fan, Y. H., Ren, H. W., & Wu, S. T. (2005). *OPTICS EXPRESS*, 13(11), 4141–4147.
- [18] Hidetoshi, N., Yoichi, T., & Kuniaki, S. (2006). Patent US No. 20060007302 A1.